

## Contents

|                                                     |           |
|-----------------------------------------------------|-----------|
| <b>Preface</b>                                      | <b>ix</b> |
| <b>1 Correlation Risk in Finance</b>                | <b>1</b>  |
| 1.1 Correlation Risk in Pricing and Risk Management | 1         |
| 1.2 Implied vs Realized Correlation                 | 3         |
| 1.3 Bottom-up vs Top-down Models                    | 4         |
| 1.4 Copula Functions                                | 4         |
| 1.5 Spatial and Temporal Dependence                 | 5         |
| 1.6 Long-range Dependence                           | 5         |
| 1.7 Multivariate GARCH Models                       | 7         |
| 1.8 Copulas and Convolution                         | 8         |
| <b>2 Copula Functions: The State of the Art</b>     | <b>11</b> |
| 2.1 Copula Functions: The Basic Recipe              | 11        |
| 2.2 Market Co-movements                             | 14        |
| 2.3 Delta Hedging Multivariate Digital Products     | 16        |
| 2.4 Linear Correlation                              | 19        |
| 2.5 Rank Correlation                                | 20        |
| 2.6 Multivariate Spearman's Rho                     | 22        |
| 2.7 Survival Copulas and Radial Symmetry            | 23        |
| 2.8 Copula Volume and Survival Copulas              | 24        |
| 2.9 Tail Dependence                                 | 27        |
| 2.10 Long/Short Correlation                         | 27        |
| 2.11 Families of Copulas                            | 29        |
| 2.11.1 Elliptical Copulas                           | 29        |
| 2.11.2 Archimedean Copulas                          | 31        |
| 2.12 Kendall Function                               | 33        |
| 2.13 Exchangeability                                | 34        |
| 2.14 Hierarchical Copulas                           | 35        |
| 2.15 Conditional Probability and Factor Copulas     | 39        |
| 2.16 Copula Density and Vine Copulas                | 42        |
| 2.17 Dynamic Copulas                                | 45        |
| 2.17.1 Conditional Copulas                          | 45        |
| 2.17.2 Pseudo-copulas                               | 46        |

|          |                                                                   |            |
|----------|-------------------------------------------------------------------|------------|
| <b>3</b> | <b>Copula Functions and Asset Price Dynamics</b>                  | <b>49</b>  |
| 3.1      | The Dynamics of Speculative Prices                                | 49         |
| 3.2      | Copulas and Markov Processes: The DNO approach                    | 51         |
| 3.2.1    | The * and ★ Product Operators                                     | 52         |
| 3.2.2    | Product Operators and Markov Processes                            | 55         |
| 3.2.3    | Self-similar Copulas                                              | 58         |
| 3.2.4    | Simulating Markov Chains with Copulas                             | 62         |
| 3.3      | Time-changed Brownian Copulas                                     | 63         |
| 3.3.1    | CEV Clock Brownian Copulas                                        | 64         |
| 3.3.2    | VG Clock Brownian Copulas                                         | 65         |
| 3.4      | Copulas and Martingale Processes                                  | 66         |
| 3.4.1    | C-Convolution                                                     | 67         |
| 3.4.2    | Markov Processes with Independent Increments                      | 75         |
| 3.4.3    | Markov Processes with Dependent Increments                        | 78         |
| 3.4.4    | Extracting Dependent Increments in Markov Processes               | 81         |
| 3.4.5    | Martingale Processes                                              | 83         |
| 3.5      | Multivariate Processes                                            | 86         |
| 3.5.1    | Multivariate Markov Processes                                     | 86         |
| 3.5.2    | Granger Causality and the Martingale Condition                    | 88         |
| <b>4</b> | <b>Copula-based Econometrics of Dynamic Processes</b>             | <b>91</b>  |
| 4.1      | Dynamic Copula Quantile Regressions                               | 91         |
| 4.2      | Copula-based Markov Processes: Non-linear Quantile Autoregression | 93         |
| 4.3      | Copula-based Markov Processes: Semi-parametric Estimation         | 99         |
| 4.4      | Copula-based Markov Processes: Non-parametric Estimation          | 108        |
| 4.5      | Copula-based Markov Processes: Mixing Properties                  | 110        |
| 4.6      | Persistence and Long Memory                                       | 113        |
| 4.7      | C-convolution-based Markov Processes: The Likelihood Function     | 116        |
| <b>5</b> | <b>Multivariate Equity Products</b>                               | <b>121</b> |
| 5.1      | Multivariate Equity Products                                      | 121        |
| 5.1.1    | European Multivariate Equity Derivatives                          | 122        |
| 5.1.2    | Path-dependent Equity Derivatives                                 | 125        |
| 5.2      | Recursions of Running Maxima and Minima                           | 126        |
| 5.3      | The Memory Feature                                                | 130        |
| 5.4      | Risk-neutral Pricing Restrictions                                 | 132        |
| 5.5      | Time-changed Brownian Copulas                                     | 133        |
| 5.6      | Variance Swaps                                                    | 135        |
| 5.7      | Semi-parametric Pricing of Path-dependent Derivatives             | 136        |
| 5.8      | The Multivariate Pricing Setting                                  | 137        |
| 5.9      | H-Condition and Granger Causality                                 | 137        |
| 5.10     | Multivariate Pricing Recursion                                    | 138        |
| 5.11     | Hedging Multivariate Equity Derivatives                           | 141        |
| 5.12     | Correlation Swaps                                                 | 144        |
| 5.13     | The Term Structure of Multivariate Equity Derivatives             | 147        |
| 5.13.1   | Altiplanos                                                        | 148        |
| 5.13.2   | Everest                                                           | 150        |
| 5.13.3   | Spread Options                                                    | 150        |

|                                                                            |            |
|----------------------------------------------------------------------------|------------|
| <b>6 Multivariate Credit Products</b>                                      | <b>153</b> |
| 6.1 Credit Transfer Finance                                                | 153        |
| 6.1.1 Univariate Credit Transfer Products                                  | 154        |
| 6.1.2 Multivariate Credit Transfer Products                                | 155        |
| 6.2 Credit Information: Equity vs CDS                                      | 158        |
| 6.3 Structural Models                                                      | 160        |
| 6.3.1 Univariate Model: Credit Risk as a Put Option                        | 160        |
| 6.3.2 Multivariate Model: Gaussian Copula                                  | 161        |
| 6.3.3 Large Portfolio Model: Vasicek Formula                               | 163        |
| 6.4 Intensity-based Models                                                 | 164        |
| 6.4.1 Univariate Model: Poisson and Cox Processes                          | 165        |
| 6.4.2 Multivariate Model: Marshall–Olkin Copula                            | 165        |
| 6.4.3 Homogeneous Model: Cuadras Augé Copula                               | 167        |
| 6.5 Frailty Models                                                         | 170        |
| 6.5.1 Multivariate Model: Archimedean Copulas                              | 170        |
| 6.5.2 Large Portfolio Model: Schönbucher Formula                           | 171        |
| 6.6 Granularity Adjustment                                                 | 171        |
| 6.7 Credit Portfolio Analysis                                              | 172        |
| 6.7.1 Semi-supervised Cluster Analysis: <i>K-means</i>                     | 172        |
| 6.7.2 Unsupervised Cluster Analysis: Kohonen Self-organizing Maps          | 174        |
| 6.7.3 (Semi-)unsupervised Cluster Analysis: Hierarchical Correlation Model | 175        |
| 6.8 Dynamic Analysis of Credit Risk Portfolios                             | 176        |
| <b>7 Risk Capital Management</b>                                           | <b>181</b> |
| 7.1 A Review of Value-at-Risk and Other Measures                           | 181        |
| 7.2 Capital Aggregation and Allocation                                     | 185        |
| 7.2.1 Aggregation: <i>C</i> -Convolution                                   | 187        |
| 7.2.2 Allocation: Level Curves                                             | 189        |
| 7.2.3 Allocation with Constraints                                          | 191        |
| 7.3 Risk Measurement of Managed Portfolios                                 | 193        |
| 7.3.1 Henriksson–Merton Model                                              | 195        |
| 7.3.2 Semi-parametric Analysis of Managed Funds                            | 200        |
| 7.3.3 Market-neutral Investments                                           | 201        |
| 7.4 Temporal Aggregation of Risk Measures                                  | 202        |
| 7.4.1 The Square-root Formula                                              | 203        |
| 7.4.2 Temporal Aggregation by <i>C</i> -convolution                        | 203        |
| <b>8 Frontier Issues</b>                                                   | <b>207</b> |
| 8.1 Lévy Copulas                                                           | 207        |
| 8.2 Pareto Copulas                                                         | 210        |
| 8.3 Semi-martingale Copulas                                                | 212        |
| <b>A Elements of Probability</b>                                           | <b>215</b> |
| A.1 Elements of Measure Theory                                             | 215        |
| A.2 Integration                                                            | 216        |
| A.2.1 Expected Values and Moments                                          | 217        |
| A.3 The Moment-generating Function or Laplace Transform                    | 218        |

viii Contents

---

|          |                                                |            |
|----------|------------------------------------------------|------------|
| A.4      | The Characteristic Function                    | 219        |
| A.5      | Relevant Probability Distributions             | 219        |
| A.6      | Random Vectors and Multivariate Distributions  | 224        |
| A.6.1    | The Multivariate Normal Distribution           | 225        |
| A.7      | Infinite Divisibility                          | 226        |
| A.8      | Convergence of Sequences of Random Variables   | 228        |
| A.8.1    | The Strong Law of Large Numbers                | 229        |
| A.9      | The Radon–Nikodym Derivative                   | 229        |
| A.10     | Conditional Expectation                        | 229        |
| <b>B</b> | <b>Elements of Stochastic Processes Theory</b> | <b>231</b> |
| B.1      | Stochastic Processes                           | 231        |
| B.1.1    | Filtrations                                    | 231        |
| B.1.2    | Stopping Times                                 | 232        |
| B.2      | Martingales                                    | 233        |
| B.3      | Markov Processes                               | 234        |
| B.4      | Lévy Processes                                 | 237        |
| B.4.1    | Subordinators                                  | 240        |
| B.5      | Semi-martingales                               | 240        |
|          | <b>References</b>                              | <b>245</b> |
|          | <b>Extra Reading</b>                           | <b>251</b> |
|          | <b>Index</b>                                   | <b>259</b> |